Bootstrapping and Learning PDFA in Data Streams

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UNIVERSITAT POLITÈCNICA DE CATALUNYA

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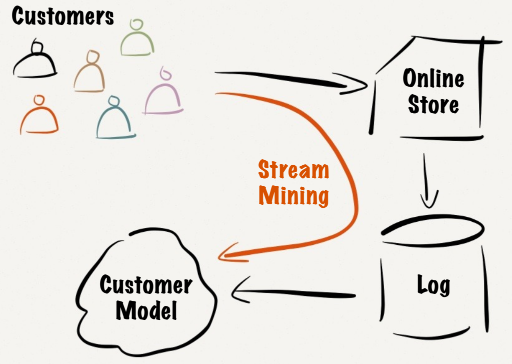
Example Application: Web User Modeling



"Wish List"

- Process examples *as fast as they arrive* (10^5 per sec. or more)
- Use *small amount of memory* (must fit into machine's main memory)
- Detect *changes* in customer behavior and *adapt* the model accordingly

Other Applications: Process Mining, Biological Models (DNA and aminoacid sequences)

Outline

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The Data Streams Algorithmic Model

An algorithm receives an infinite stream $x_1, x_2, \ldots, x_t, \ldots$ from some domain X and must:

- Make only one pass over the data and process each item in time $O(1)$
- At every time t use sublinear memory (e.g. $O(\log t)$, $O(\sqrt{t})$)
- Adapt to possible "changes" in the data

It is a theoretically challenging model useful for applications:

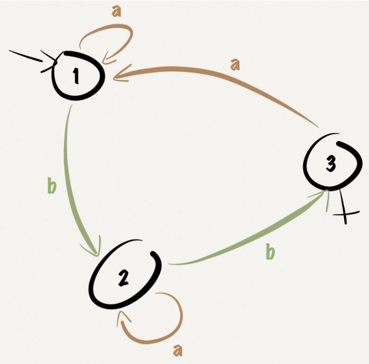
- Originated in the algorithmics community
- Realistic for Data Mining and Machine Learning tasks in real-time
- Feasible way to deal with Big Data problems

When studying learning problems with streaming data:

- In the worst case setting it resembles Gold's model (with algorithmic constraints)
- But we consider a PAC-style scenario where:
  - x_t are all independent and generated from a distribution D_t
  - the sequence of distributions $D_1, D_2, \ldots, D_t, \ldots$ either *changes very slowly* or presents only *abrupt changes but very rarely*

Hypothesis Class: PDFA

Probabilistic Deterministic Finite Automata = DFA + Probabilities



Transition/Stop probabilities

q	$p_q(a)$	$p_q(b)$	$p_q(\xi)$
1	0.3	0.7	0.0
2	0.5	0.5	0.0
3	0.8	0.0	0.2

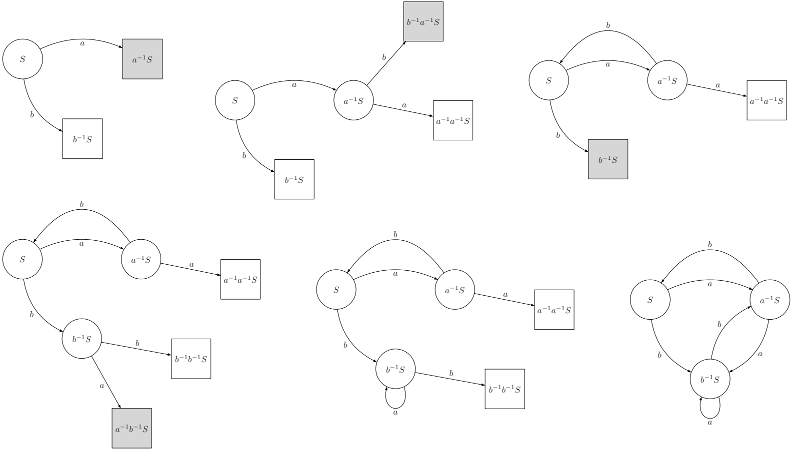
Parameters

- n (states)
- $|\Sigma|$ (alphabet)
- L (expected length)
- μ (distinguishability, L_∞)

$\mu = \min_{q \neq q'} \max_{x \in \Sigma^\star} |D_q(x) - D_{q'}(x)|$

State Merge/Split Algorithm

Usual approach to PDFA learning [Carrasco–Oncina '94, Ron et al. '98, Clark–Thollard '04, Palmer–Goldberg '05, Castro–Gavaldà '08, etc.]



Statistical tests

$$S \not\approx a^{-1}S$$
$$S \approx b^{-1}a^{-1}S$$
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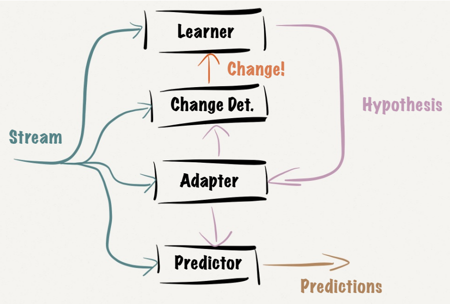
Description of the Algorithm

System Architecture



Learner Module

initialize H with safe q_λ;
foreach $\sigma \in \Sigma$ **do**
    add a candidate q_σ to H;
    schedule insignificance and similiarity tests for q_σ;
foreach *string* x_t *in the stream* **do**
    foreach *decomposition* $x_t = wz$, *with* $w, z \in \Sigma^\star$ **do**
        if q_w *is defined* **then**
            add z to $\hat{S}_w$;
            if q_w *is a candidate* **and** $|\hat{S}_w|$ *is large enough* **then** call SimilarityTest(q_w, δ);
    foreach *candidate* q_w **do**
        if *it is time to test insignificance of* q_w **then**
            if $|\hat{S}_w|$ *is too small* **then** declare q_w insignificant;
            else schedule another insignificance test for q_w;
    if H *has more than* n *safes* **or** *there are no candidates left* **then**
    return H;

Sample Sketches for Similarity Testing

Note: Instead of keeping a sample S_w for each state q_w, the algorithm keeps a *sketch* $\hat{S}_w$ of each sample

A sketch using memory $O(1/\mu)$ should be enough:

- Given samples S, S' from distributions D, D'
- Algorithm wants to test $L_\infty(D, D') = 0$ or $L_\infty(D, D') \geqslant \mu$
- In the second case, if $|D(x) - D'(x)| \geqslant \mu$ then either $D(x) \geqslant \mu$ or $D'(x) \geqslant \mu$
- It is enough to find all strings with $D(x), D'(x) = \Omega(\mu)$, of which there are $O(1/\mu)$

In our algorithm, each sketch uses a **SpaceSaving** data structure [Metwally et al. '05]:

- Uses memory $O(1/\mu)$
- Finds every string whose probability is $\Omega(\mu)$ (frequent strings)
- And approximates their probability with enough accuracy
- Easier to implement than sketches based on hash functions

Properties of the Algorithm

Streaming-specific features

- Adaptive test scheduling (decide as soon as possible)
- Similarity test based on Vapnik–Chervonenkis bound (slow similarity detection)
- Use bootstrapped confidence intervals in tests (faster convergence)

Complexity Bounds (with any reasonable test)

- Time per example $O(L)$ (expected, amortized)
- The learner reads $O(n^2|\Sigma|^2/\epsilon\mu^2)$ examples (in expectation)
- Memory usage is $O(n|\Sigma|L/\mu)$ (roughly $O(\sqrt{t})$)

Outline

Testing Similarity between Probability Distributions

Goal: decide if $L_\infty(D, D') = 0$ or $L_\infty(D, D') \geqslant \mu$ from samples S, S'

Statistical Test Based on Empirical L_∞ (the "default")

- Let $\mu_\star = L_\infty(D, D')$ and compute $\hat{\mu} = L_\infty(S, S')$
- Compute Δ_l, Δ_u such that $\hat{\mu} - \Delta_l \leqslant \mu_\star \leqslant \hat{\mu} + \Delta_u$ holds w.h.p.
- If $\hat{\mu} - \Delta_l > 0$ decide $D \neq D'$
- If $\hat{\mu} + \Delta_u < \mu$ decide $D = D'$
- Else, wait for more examples

Problem: asymmetry — deciding *dissimilarity* is easier that deciding *similarity*

- When $D \neq D'$ will decide correctly w.h.p. when $|S|, |S'| \approx 1/\mu_\star^2$
- When $D = D'$ will decide correctly w.h.p. when $|S|, |S'| \approx 1/\mu^2$

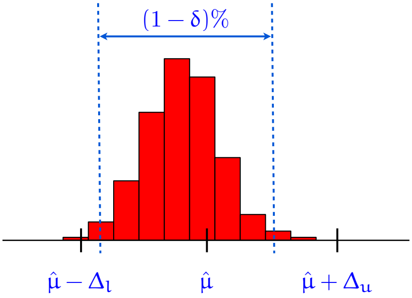
In the later we are always competing against the *worst case* $L_\infty(D, D') = \mu$

Enter the Bootstrap

- In the test I just described there is another *worst case* assumption — the confidence interval $\mu_\star \leqslant \hat{\mu} + \Delta_u$ must hold *for any D and D'*
- *But* it may be the case that for some D, certifying that $S, S' \sim D$ come from the same distribution is *easier*
- The *bootstrap* is widely used in statistics for computing *distribution dependent* confidence intervals (among many other things)

Basic Idea

- Suppose we have r different samples $S_{(1)}, \ldots, S_{(r)} \sim D$
- Compute distances $\hat{\mu}_i = \mathrm{L}_\infty(S_{(i)}, S'_{(i)})$
- Use them to compute a histogram of the distribution of $\hat{\mu}$



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Bootstrapped Confidence Intervals

- Given a sample S, obtain other samples $\tilde{S}_{(i)}$ by sampling from S uniformly with replacement
- Sort estimates increasingly $\tilde{\mu}_1 \leqslant \ldots \leqslant \tilde{\mu}_r$
- Say that $\mu_\star \leqslant \tilde{\mu}_{\lceil(1-\delta)r\rceil}$ with prob. $\geqslant 1-\delta$

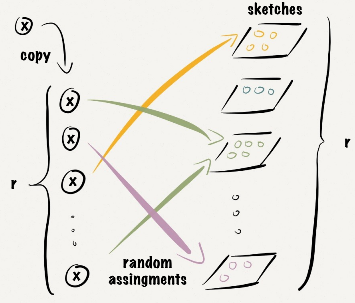
Bootstrapped Confidence Intervals in Data Streams

Question: Do you need to *store the full sample* to do bootstrap resampling?

Answer: No, if you can *test from sketched data*

The Bootstrap Sketch

- Keep r copies of the sketch you use for testing (e.g. **SpaceSaving**)
- For each item x_t in the stream, randomly insert r copies of x_t into the r sketches
- Comparing each pair $\tilde{S}_{(i)}, \tilde{S}'_{(j)}$ can obtain r^2 approximations $\tilde{\mu}_{i,j}$
- Choosing r involves a trade-off between accuracy and memory



In theory can prove bound (asymptotically) comparable to Vapnik–Chervonenkis

In practice assuming $\mu_\star \leqslant \tilde{\mu}_{\lceil(1-\delta)r^2\rceil}$ gives accurate and statisically efficient similarity test

Experimental Results for Learner

- Prototype written in C++ and Boost, run in this laptop
- Evaluated with Reber Grammar (typical Grammatical Inference benchmark)
  - $|\Sigma| = 5$, $n = 6$, $\mu = 0.2$, $L \approx 8$
- Compared VC and Bootstrap ($r = 10$) based tests

	Examples	Memory (MiB)	Time/item (ms)
Hoeffding	57617	6.1	0.05
Bootstrap	23844	53.7	1.2

Outline

What if n and μ are unknown (or change)?

Want to design strategy for *fast and accurate* parameter estimation

Parameter Search Algorithm

$n \leftarrow 2$, $\mu \leftarrow 1/8$;
while true do
  $H \leftarrow \texttt{Learner}(n, \mu)$;
  if $|H| < n$ **then** $\mu \leftarrow \mu/8$;
  else $n \leftarrow 2n$;
  if $n > (1/\mu)^{1/3}$ **then** $\mu \leftarrow \mu/8$;

Complexity Bounds

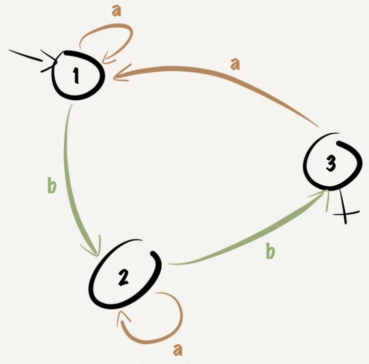
- Needs only $O(\log(n_\star/\mu_\star^{1/3}))$ calls to **Learner**
- In expectation will read $O(n_\star^6 |\Sigma|^2/\varepsilon\mu_\star^2)$ elements
- Memory usage grows like $O(t^{2/3})$

Note: can tweak parameters to trade-off convergence speed and memory usage

Adapting the Hypothesis to Changes

Adapter block — Once the structure is known...

- Estimating probabilities is easy
- Estimations can be adapted to changes (e.g. moving average)



Transition/Stop probabilities

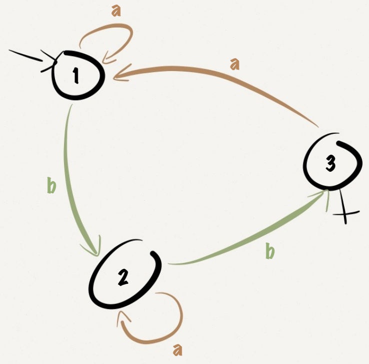
$S = \{abb, baab, bbaabb\}$

q	$p_q(a)$	$p_q(b)$	$p_q(\xi)$
1	2/6	4/6	0/6
2	2/6	4/6	0/6
3	1/4	0/4	3/4

But, sometimes the current structure is not good anymore

Detecting Structural Changes

Idea: "change" is *difficult to define* in general, focus on changes explained in terms of *structure*



- Given a *PDFA*, compute the expected number of times each state is visited when generating a string
- Given a *sample*, compute the average number of times strings hit any state
- If there is a *significant difference*, conclude the structure has changed

$$S = \{abb, baab, bbaabb\}$$

h_1	h_2	h_3
6/3	6/3	4/3

- Restart structure learning when a change is detected
- Adapting probabilities may be enough, but re-learning does no damage

Outline

Conclusion

Summary of Contributions

- Adaptation of state-merging paradigm to streaming data
- Fast convergence achieved by:
  - adaptive test scheduling
  - better similarity testing
  - efficient parameter search
- Use of sketching algorithms for implementing the bootstrap and reducing memory usage

Future Work

- Deploy real system and exploit parallelization oportunities
- Develop further similarity tests based on the bootstrap
- Adapt other GI algorithms to the data streams framework

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